Introduction

The word “trigonometry” is derived from two Greek words *trigonon*, meaning triangle and *metria*, means measurement. Trigonometry is the study of triangles. Angles and their measure are fundamental to the study of trigonometry. We begin our study of trigonometry by looking at angles and methods for measuring them.

Angles

We begin with basic terminology. Two distinct points A and B determine a line called *line AB*, denoted \( \overline{AB} \). The portion of the line between A and B, including points A and B themselves is the *line segment AB*, denoted \( \overline{AB} \). The portion of \( \overline{AB} \) that starts at A and continues through B and on is called *ray AB*, denoted \( \overrightarrow{AB} \). The point A is the endpoint of the ray.

An *angle* is a figure formed by two rays with a common endpoint. The endpoint is called the *vertex* of the angle. Angles can be named many ways. A vertex is sometimes sufficient, but can be ambiguous. Greek letters such as \( \theta \) (theta), \( \alpha \) (alpha), \( \beta \) (beta) are commonly used. The following are various ways to name the angle shown below.

<table>
<thead>
<tr>
<th>Angle</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle B:</td>
<td>( \angle B )</td>
</tr>
<tr>
<td>Angle ( \theta ):</td>
<td>( \theta )</td>
</tr>
<tr>
<td>Angle: ABC</td>
<td>( \angle ABC )</td>
</tr>
</tbody>
</table>
Angles can be formed by rotating a ray around its endpoint. The ray in its initial position is called the initial side of the angle. The location of the ray after the rotation is the terminal side of the angle. The direction of rotation is denoted by a positive or negative value. If the rotation is counterclockwise, the angle is positive. If the rotation is clockwise, the angle is negative. The direction of rotation is indicated by an arrow as shown.

Measuring Angles Using Degrees

Angles are measured by determining the amount of rotation from the initial side to the terminal side. Developed by the Babylonians over 4000 years ago, the most common unit for measuring angles today is the degree (symbolized by a small superscript circle, °). By definition, one complete rotation of a ray is 360°. Based on this, 1° = \(\frac{1}{360}\) of a rotation. Think of a circle being divided into 360 equal parts.

An angle measuring exactly 90° is a right angle. An angle measuring between 0° and 90° is called an acute angle. And angle measuring more than 90° but less than 180° is an obtuse angle. An angle whose measure is exactly 180° is called a straight angle.
Two positive angles are said to be **complimentary** if the sum of the measures is $90^\circ$. If the sum of the measures of two positive angles is $180^\circ$, the angles are supplementary.

**Example 1**
Find the measures of angle $\theta$:

![Diagram](image1)

**Example 2**
Find the measures of angle $\theta$:

![Diagram](image2)

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**Standard Position**

An angle is in standard position if
- Its vertex is at the origin of a rectangular coordinate system, and
- Its initial side lies along the positive $x$-axis.

![Diagram](image3)

Standard position allows us to examine positive and negative angles as well as angles of any size. Angles are further classified by the quadrant in which the terminal side lands. An angle in standard position is said to lie in the quadrant in which its terminal side lies. For example, an angle whose terminal side sits in the second quadrant is a second quadrant angle. An acute angle has a terminal side in quadrant I. An obtuse angle has a terminal side in quadrant II. More importantly, an angle in standard position whose terminal side sits along the $x$-axis or $y$-axis is called a **quadrantal angle**. Such angles with measures of $0^\circ$, $90^\circ$, $180^\circ$, $270^\circ$, $360^\circ$, and so on, are examples of quadrantal angles.
Example 1
- A second quadrant angle
- An obtuse angle

Example 2
- A first quadrant angle
- An acute angle

Example 3
- A quadrant angle
- Neither acute nor obtuse

Example 4
- A fourth quadrant angle
- An angle whose measure is negative

Example 5: Draw each angle in standard position
- a. $210^0$
- b. $-225^0$
- c. $270^0$
- d. $-150^0$
Coterminal Angles

A complete rotation of a ray results in an angle measuring 360°. By continuing the rotation, angles of measure larger than 360° can be produced. Because of this, it is possible for two angles to have the same terminal side. Two angles with the same initial and terminal sides are called coterminal angles. There are many possible coterminal angles to any given angle.

An angle, \( \theta \), is coterminal to angles \( \theta + 360^\circ k \) where \( k \) is any integer.

Example 1

270° and -90° are coterminal angles

Example 2: Draw each angle in standard position. Find one positive and one negative coterminal angle.

a. 300°

b. 20°

c. 520°

d. -150°
Measuring Angles Using Radians

Another way to measure angle is in radians. Radian measure allows us to use real numbers as measures of angles. We begin with a circle of radius \( r \). An angle whose vertex is the center of the circle is called a central angle. The central angle intercepts and arc along the circle. The arc length of the circle cut by central angle is denoted by \( s \). The radian measure of a central angle is the ratio of the arc length to the radius: \( \theta \) radians = \( \frac{\text{arclength}}{\text{radius}} \)

\[ \theta = \frac{s}{r} \]

To get a sense of the size of one radian, consider this: 1 radian is the measure of the central angle needed to cut an arc equal to the radius.

**Example:** Use the definition of a radian to find the measure of each of the following central angles in radian form.

**Workspace**

1.

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Introduction to Trigonometry

Angles and Their Measure

2. e) \[ \theta = 2.8 \]

3. f) \[ \theta = 2 \]

4. \[ \theta = \sqrt{26} \]

5. \[ \theta = \frac{10}{7} \]

6. \[ \theta = 2.8 \text{ rad} \]

Workspace
Relationship Between Degrees and Radians

We can use the radian measure formula in an alternated for to find the length of the arch of a circle.

Alternate form: \( s = r \theta \)

**Caution:** When using this formula, you must remember that \( \theta \) is an angle measured in radians!

What do we do if the angle is given in degree form? The angle must be converted. How? We compare the number of degrees and the number of radians in one complete rotation. We know that \( 360^\circ \) is the degree measure of one rotation. Can we find the equivalent radian measure? In order to do this, we need to know the arc length of one complete rotation. This is the equivalent of the circumference of a circle. Recall from geometry: \( C = 2\pi r \). Therefore the arc length of a circle of radius \( r \) is \( 2\pi r \). Applying the definition of a radian we have the following:

\[
1 \text{ revolution} = 360^\circ = \frac{s}{r} = \frac{2\pi r}{r} = \frac{2\pi \theta}{\theta} = 2\pi
\]

\( 360^\circ = 2\pi \text{ radians} \)

Dividing both sides by 2: \( 180^\circ = \pi \)

We can use this relationship to develop a method for converting between degrees and radians as follows:

\[
1 \text{ radian} = \frac{180^\circ}{\pi} \quad \text{or} \quad 1^\circ = \frac{\pi}{180^\circ}
\]
Converting Between Degrees and Radians

- To convert to radians: multiply the degree measure by \( \frac{\pi}{180} \)
- To convert to degrees: multiply the radian measure by \( \frac{180}{\pi} \)

**Example 1:** Convert to radians

1. \( 15^\circ \) __________
2. \( 30^\circ \) __________
3. \( 200^\circ \) __________

**Example 2:** Convert to degrees

1. \( \frac{2\pi}{3} \) __________
2. \( \frac{5\pi}{6} \) __________
3. \( 5 \) __________
1. Complete the chart by converting each angle. Express answers in exact form.

<table>
<thead>
<tr>
<th>Degrees</th>
<th>Radians</th>
<th>Work space</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. 120°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>$\frac{7\pi}{4}$</td>
<td></td>
</tr>
<tr>
<td>c. 270°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. -135°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e.</td>
<td>$-\frac{5\pi}{12}$</td>
<td></td>
</tr>
<tr>
<td>f. 330°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>g.</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>h.</td>
<td>$\frac{11\pi}{3}$</td>
<td></td>
</tr>
</tbody>
</table>

2. A pebble stuck to a bicycle wheel with a diameter of 26 inches rotates through an angle of 160°. To the nearest tenth of an inch, approximately how far did the pebble travel?
3. Draw each angle in standard position and find at least two coterminal angles.

a. $210^0$

b. $-100^0$

c. $\frac{2\pi}{5}$

d. $1090^0$

e. $-310^0$

f. $\frac{2\pi}{3}$

g. $-225^0$

h. $-\pi$